EFFICIENT ALGORITHM FOR LOCALIZING 3D NARROWBAND MULTIPLE SOURCES USING A Y-SHAPED ARRAY

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ABSTRACT

This paper proposes an efficient 3D source localization algorithm using 3 uniform linear subarrays. The proposed algorithm replaces the 3D search required in conventional 3D MUSIC with 3 1D searches, and thus reduces the computational burden. An estimate of the 1D incident angle obtained from a subarray under the far field assumption satisfies the nonlinear algebraic equation of the true source bearing angle, elevation angle, and range. The proposed algorithm estimates source location by solving 3 algebraic equations obtained from 3 subarrays. When comparing 3D MUSIC spectrums of the estimated source locations, the proposed algorithm is able to solve the pairing problem in multiple sources localization.

1. INTRODUCTION

Passive source localization using an array of sensors, that is, the estimation of position coordinates(range, azimuth angle, elevation angle) of noisy sources in 3D space, is an important topic in various signal processing fields such as radar, sonar, speech, communication, etc. In far field source localization, a signal wavefront can be assumed to be planar and, as a result, only the incident angles of the signal(azimuth and/or elevation) can be estimated[1]. However, for near field source localization, the range to the source as well as the incident angles must be estimated because the wavefront of the source signal is spherical[2]-[8].

In 2D near field source localization, where a source location is represented as a range and a bearing angle, 2D MUSIC requires a 2D grid search on the range and the bearing angle to identify the peak of the MUSIC spectrum and, as a consequence, is computationally inefficient[2]. Weiss *et al.* estimated source locations using a polynomial rooting method thereby reducing the computational burden to some extent, however, the algorithm still requires bulk computations[3]. Starer *et al.*[4] and Lee *et al.*[5] proposed path-following algorithms in which 1D paths are setup and the peaks of 2D MUSIC spectrum are then identified by following these paths. In the latter, the paths to the peak of the 2D MUSIC are algebraically setup from the the estimates of 1D far field bearing angles. Thus the algorithm replaces a 2D search with 2 1D searches, one for estimating the far field bearing angles and the other for following the paths. Lee *et al.* also proposed a nonlinear triangulation ranging algorithm using 2 subarrays. This algorithm, which requires 2 1D searches, reduces the estimation bias compared with the conventional triangulation ranging algorithm[6].

Although researchers have shown much interest about 3D source localization, the problem has not been widely treated in literature. Hung *et al.*[7] proposed an algorithm which is a 3D extension of the algorithm proposed by Weiss *et al.* In this algorithm, the pairing problem is not addressed for multiple sources with coincident coordinates. Challa *et al.* proposed an algorithm using cumulant processing and ESPRIT[8]. Although it solves the pairing problem arising in a multiple source case, it still requires a large amount of computation and is applicable only for non-Gaussian source signals.

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Accordingly this paper proposes an efficient 3D source localization algorithm using 3 uniform linear arrays(ULAs) of sensors. The estimate of the 1D incident angle, obtained from a subarray under the far field assumption, satisfies the nonlinear algebraic equation of the true source bearing angle, elevation angle, and range. Thus, the source location can be estimated by simultaneously solving 3 equations obtained from 3 subarrays and then performing a local search using the solution as the initial point. The proposed algorithm replaces the 3D search required in the conventional 3D MUSIC with 3 1D searches which are required for estimating the 1D incident angles for the 3 subarrays, thereby reducing the computational complexity. Furthermore, the proposed algorithm can be adopted to multiple source cases using a simple pair-matching method in which the 3D MUSIC spectrum values of all possible combinations of the 1D incident angles are compared to avoid any *ghost* estimates.

Section II introduces the problem considered in this paper. Section III presents the proposed algorithm, then some numerical simulations are performed in section IV to verify the performance of the proposed algorithm.

2. PROBLEM FORMULATION

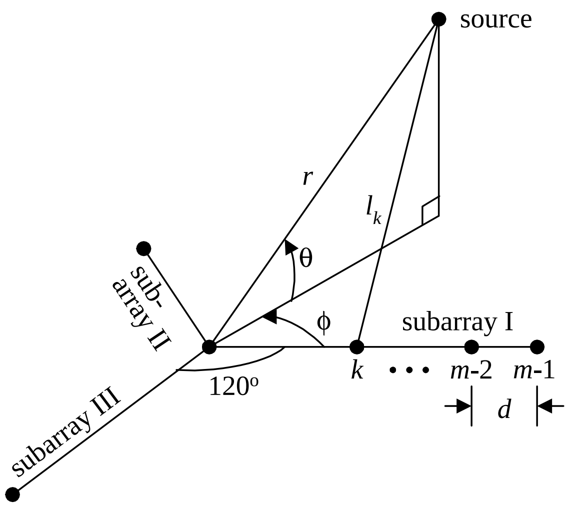

Fig. 1. Geometry of source and sensor array.

The geometry of a source and a sensor array is shown in Fig. 1. A sensor array consists of 3 ULAs lying in the same plane, and each of the 3 ULAs consists of m identical sensors equally spaced at a distance of d. As shown in the figure, a near field source is located at range r measured from the center of the array, azimuth angle ϕ measured counterclockwise from subarray I, and elevation angle θ measured from the plane where the sensor array lies. 3D MUSIC then identifies the estimate of the source location which maximizes the MUSIC spectrum

$$P(r,\phi,\theta) = \frac{1}{a^H(r,\phi,\theta)E_N E_N^H a(r,\phi,\theta)} \tag{1}$$

where the superscript H denotes a complex conjugate transposition, and E_N is the noise subspace of the sensor output covariance matrix. $a(r,\phi,\theta)$ is the steering vector whose kth element is given as

$$a^{(k)}(r,\phi,\theta) = \exp\{-j\omega\tau_k(r,\phi,\theta)\} \tag{2}$$

where ω is the carrier frequency of the source signal. The difference $\tau_k(r,\phi,\theta)$ in time between the signal arriving at the kth sensor and the signal arriving at the reference sensor in the centre of the sensor array is given by

$$\tau_k(r,\phi,\theta) = \frac{1}{c_0}(r - l_k) \tag{3}$$

where c_0 is the wave propagation speed and l_k is the distance between the source and the kth sensor. However, the 3D MUSIC is computationally inefficient because it requires an exhaustive 3D grid search, followed by a local search, to identify the peak of eqn. 1.

3. PROPOSED ALGORITHM

This section proposes an efficient algorithm for locating the peak of the 3D MUSIC spectrum. If a near field source is assumed to be in a far field, the time delays between the signal arriving at adjacent sensors in one of the subarrays are the same as $d/c_0 \cos\xi$ where ξ is the far field incident angle of the underlying far field source. However, since the wavefront of a near field source is spherical, true time delays are not identical. Thus, for subarray I, the estimate of the far field incident angle can minimize the sum of squared errors of the constant time delay $d/c_0 \cos\xi$ and true time delays $(\tau_i(r,\phi,\theta) - \tau_{i-1}(r,\phi,\theta))$ as follows [5, 6]:

$$\hat{\xi}_{\mathrm{I}} = \min_{\xi} \sum_{i=1}^{m-1} \left[\frac{d}{c_0} \cos\xi - (\tau_i(r,\phi,\theta) - \tau_{i-1}(r,\phi,\theta)) \right]^2 \tag{4}$$

By solving eqn. 4 produces the following:

$$\frac{d}{c_0} \cos\hat{\xi}_{\mathrm{I}} = \frac{\tau_{m-1}(r,\phi,\theta) - \tau_0(r,\phi,\theta)}{m-1} \tag{5}$$

Substituting eqn. 3 into eqn. 5 yields

$$D\cos\hat{\xi}_{\mathrm{I}} = r - l_{m-1} \tag{6}$$

where $D = (m-1)d$ is the aperture of the subarrays and l_{m-1} is the distance between the last sensor of subarray I and the source.

$$l_{m-1} = \sqrt{D^2 + r^2 - 2rD\cos\phi\cos\theta} \tag{7}$$

Substituting eqn. 7 into eqn. 6, produces

$$\cos\theta\cos\phi = \cos\hat{\xi}_{\mathrm{I}} + \frac{1}{2}(1-\cos^2\hat{\xi}_{\mathrm{I}})\frac{D}{r} \tag{8}$$

Note that, eqn. 8 gives the algebraic relation between the near field range r, the azimuth angle ϕ, and the elevation angle θ for a given far field incident angle $\hat{\xi}_{\mathrm{I}}$. Other algebraic equations can be similarly obtained for subarray II and III as

$$\cos\theta\cos(\phi - 120^\circ) = \cos\hat{\xi}_{\mathrm{II}} + \frac{1}{2}(1-\cos^2\hat{\xi}_{\mathrm{II}})\frac{D}{r} \tag{9}$$

$$\cos\theta\cos(\phi + 120^\circ) = \cos\hat{\xi}_{\mathrm{III}} + \frac{1}{2}(1-\cos^2\hat{\xi}_{\mathrm{III}})\frac{D}{r} \tag{10}$$

The matrix representation of eqns. 8, 9, and 10 is

$$\begin{pmatrix} \cos\hat{\xi}_{\mathrm{I}} \\ \cos\hat{\xi}_{\mathrm{II}} \\ \cos\hat{\xi}_{\mathrm{III}} \end{pmatrix} = \begin{pmatrix} -\frac{1}{2}(1-\cos^2\hat{\xi}_{\mathrm{I}}) & 1 & 0 \\ -\frac{1}{2}(1-\cos^2\hat{\xi}_{\mathrm{II}}) & -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{1}{2}(1-\cos^2\hat{\xi}_{\mathrm{III}}) & -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix} \begin{pmatrix} \frac{D}{r} \\ \cos\phi\cos\theta \\ \sin\phi\cos\theta \end{pmatrix} \tag{11}$$

The initial estimate of the source location is obtained by solving eqn. 11 for r/D, $\cos\phi$, and $\cos\theta$.

$$r = D/\alpha \tag{12}$$

$$\cos\theta = \sqrt{\beta^2 + \gamma^2} \tag{13}$$

$$\cos\phi = \beta/\cos\theta \tag{14}$$

where α, β, and γ are represented as functions of the far field incident angles of $\hat{\xi}_{\mathrm{I}}$, $\hat{\xi}_{\mathrm{II}}$, and $\hat{\xi}_{\mathrm{III}}$.

$$\alpha = -2\frac{\cos\hat{\xi}_{\mathrm{I}} + \cos\hat{\xi}_{\mathrm{II}} + \cos\hat{\xi}_{\mathrm{III}}}{\sin^2\hat{\xi}_{\mathrm{I}} + \sin^2\hat{\xi}_{\mathrm{II}} + \sin^2\hat{\xi}_{\mathrm{III}}} \tag{15}$$

$$\beta = \frac{\cos\hat{\xi}_{\mathrm{I}}\sin^2\hat{\xi}_{\mathrm{II}} + \cos\hat{\xi}_{\mathrm{I}}\sin^2\hat{\xi}_{\mathrm{III}} - \cos\hat{\xi}_{\mathrm{II}}\sin^2\hat{\xi}_{\mathrm{I}} - \cos\hat{\xi}_{\mathrm{III}}\sin^2\hat{\xi}_{\mathrm{I}}}{\sin^2\hat{\xi}_{\mathrm{I}} + \sin^2\hat{\xi}_{\mathrm{II}} + \sin^2\hat{\xi}_{\mathrm{III}}} \tag{16}$$

$$\gamma = \frac{\cos\hat{\xi}_{\mathrm{I}}\sin^2\hat{\xi}_{\mathrm{II}} + \cos\hat{\xi}_{\mathrm{III}}\sin^2\hat{\xi}_{\mathrm{I}} + 2\cos\hat{\xi}_{\mathrm{III}}\sin^2\hat{\xi}_{\mathrm{II}} - \cos\hat{\xi}_{\mathrm{I}}\sin^2\hat{\xi}_{\mathrm{III}} - \cos\hat{\xi}_{\mathrm{II}}\sin^2\hat{\xi}_{\mathrm{I}} - 2\cos\hat{\xi}_{\mathrm{II}}\sin^2\hat{\xi}_{\mathrm{III}}}{\sqrt{3}(\sin^2\hat{\xi}_{\mathrm{I}} + \sin^2\hat{\xi}_{\mathrm{II}} + \sin^2\hat{\xi}_{\mathrm{III}})} \tag{17}$$

Table 1. Mean and variance of the distance between the peak of the 3D MUSIC spectum and the estimate of the proposed algorithm without a local search.

		0dB	1dB	2dB
Range	Mean	0.0734	0.0498	0.0346
(r/D)	Var.	0.0037	0.0014	0.0008
Azi. Ang.	Mean	0.0979	0.0784	0.0469
(Deg.)	Var.	0.0056	0.0034	0.0012
Ele. Ang.	Mean	0.1364	0.0830	0.0697
(Deg.)	Var.	0.0092	0.0035	0.0025

Thereafter the final estimate of source location is identified by a 3D local search in the 3D MUSIC spectrum using the initial estimate obtained from eqns. 12, 13,and 14 as the initial point. Although the local search is performed in 3D space, it does not require much computation as long as its initial point is close to the peak of the 3D MUSIC spectrum.

In the case of n multiple sources, a pair-matching method is required to pair the far field incident angles according to the true source locations. Here, it is assumed that the number of near field sources are properly estimated from other source number estimation algorithms. The proposed algorithm compares the 3D MUSIC spectrums from all possible combinations of far field incident angles $(\hat{\xi}_{\mathrm{I}_i}, \hat{\xi}_{\mathrm{II}_j}, \hat{\xi}_{\mathrm{III}_k}), i, j, k = 1, 2, \ldots, n$, and then chooses the n triples which produce the largest 3D MUSIC spectrum values.

The proposed algorithm can be summarized as follows:

Step 1 *Initialisation* : Estimate far field incident angles for subarrays I, II, and III by identifying the peaks of the 1D MUSIC spectrum of

$$\hat{\xi}_s = \max_{\xi_s} \frac{1}{a_s^H(\xi_s) E_{N_s} E_{N_s}^H a_s(\xi_s)}, s = \mathrm{I}, \mathrm{II}, \mathrm{III} \tag{18}$$

where subscript s indicates subarrays [1].

Step 2 *Pairing* : Choose the n triples $(\hat{\xi}_{\mathrm{I}_i}, \hat{\xi}_{\mathrm{II}_j}, \hat{\xi}_{\mathrm{III}_k}), i, j, k = 1, 2, \ldots, n$ which produce the largest 3D MUSIC spectrum values among all possible combinations of the far field incident angles obtained in Step 1.

Step 3 *Localization* : Estimate the source location by applying far field incident angles obtained in Step 2 to eqns. 12, 13, and 14.

Step 4 *Local Search* : Perform a local search to estimate the peak of the 3D MUSIC spectrum using a general optimization algorithm using the estimate obtained in Step 3 as the initial value.

4. SIMULATIONS

The proposed algorithm is applied to some simulation examples to demonstrate its performance. Each of the 3 ULAs consists of 7 identical sensors spaced half a wavelength apart. The wave propagation speed c_0 is set to 1500m/second assuming that the acoustic medium is water. The sources are assumed to emit random narrowband complex Gaussian waveforms and sensor data sample covariance matrices are computed from 256 snapshots.

In Table 1, the mean and the variance of the distance between the peak of the 3D MUSIC spectum and the estimate of the proposed algorithm without local search is shown for a source at $(r, \phi, \theta) = (5D, 45°, 45°)$ using 200-run Monte Carlo simulations. As shown in the Table, the estimate of the proposed algorithm without a local search is sufficiently close to the peak of the 3D MUSIC spectrum, consequently it is clear that a local search does not require much computation.

The proposed algorithm is applied to multiple sources where the two coordinates of the sources are identical. The sources are at $(r, \phi, \theta) = (5D, 60°, 45°)$ and $(5D, 90°, 45°)$, and at $(r, \phi, \theta) = (5D, 45°, 30°)$ and $(5D, 45°, 60°)$ for the simulations in Figs. 2 and 3, respectively. The SNR is set to 0dB and 50-run Monte Carlo simulations of the proposed algorithm without a local search are plotted in Fig. 2 and 3. In these figures, $*$ indicates the true source locations. As shown in the figures, the estimates of the proposed algorithm are clustered around the true source locations, and it is expected that more accurate estimates would be obtained through a local search. Accordingly the proposed algorithm succesfully pairs the incident angles obtained from the 3 ULAs even when the two coordinates of the source locations are identical.

Table 2. Resolution and required floating point operations of 3D MUSIC and the proposed algorithm.

Grid Size	3D MUSIC		Proposed	
$\Delta r, \Delta\cos\phi, \Delta\cos\theta$	Resol. ($\Delta\phi$)	flops	Resol. ($\Delta\phi$)	flops
(0.4D,0.1,0.1)	8°	20,266,520	12°	3,341,491
(0.2D,0.05,0.05)	6°	132,204,884	7.5°	3,529,220
(0.1D,0.025,0.025)	5.5°	990,645,963	7.5°	3,600,602
(0.05D,0.0125,0.0125)	4°	7,714,971,035	6°	3,699,617

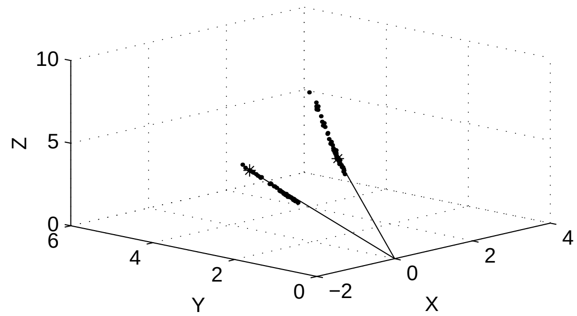

Fig. 2. Initial estimates of sources at $(r, \phi, \theta) = (5D, 60°, 45°)$ and $(5D, 90°, 45°)$.

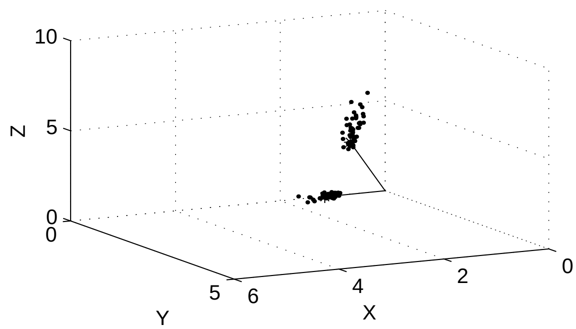

Fig. 3. Initial estimates of sources at $(r, \phi, \theta) = (5D, 45°, 30°)$ and $(5D, 45°, 60°)$.

The resolution capability of the proposed algorithm may be degraded because it uses subarrays instead of the whole sensor array to estimate far field incident angles. Table 2 lists the resolution and required floating point operations of 3D MUSIC and the proposed algorithm with respect to grid size. For the simulations, the floating point operations are obtained using the MATLAB function *flops* and the SNR is set to 10dB. Ranges and elevation angles of two sources are fixed at $5D$ and 30°, respectively. The azimuth angle of one source is fixed at 90° and the other varyed from 60° to 90°. The resolution of the algorithms is defined as the difference between the azimuth angles of the sources when the separation probability of the sources is 0.9. The grid sizes of the proposed algorithm, required in the 1D incident angle estimation, are the same as those of the azimuth angle. As listed in Table 2, the resolution capability of the proposed algorithm is about 1.5 times lower than that of 3D MUSIC. However, the flops required for 3D MUSIC are prohibitively large and grow geometrically as the gird size is reduced. Whereas, a reduction in grid size does not cause much increase in the flops for the proposed algorithm since most of the required computation is due to the eigendecomposition which is required for pair-matching. Judging from the result in Table 2, it is concluded that the resolution capability of the proposed algorithm is satisfactory.

5. CONCLUSION

The proposed algorithm vastly improves the computational efficiency of 3D localization of near field sources using 3 ULA subarrays. Instead of an exhaustive 3D grid search, the proposed algorithm requires 3 1D searches to estimate far field incident angles. Furtheremore, it is applicable to multiple sources through the use of a simple pair-matching method.

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