A NEW APPROACH FOR DESIGN SHARP FIR FILTERS USING FREQUENCY RESPONSE MASKING TECHNIQUE

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ABSTRACT

A new method to reduce the number of multipliers in the design of sharp FIR filter by frequency-response masking technique is presented. The success of the proposed method is based on a modified frequency-response masking approach where one of the subfilters in frequency-response masking approach is implemented by Interpolated Finite-Impulse Response (IFIR) technique. We have shown by example that the proposed method yields additional savings of 24% in the number of multipliers compared to the original frequency-response masking approach.

1. INTRODUCTION

Digital signal processing techniques are being applied to wireless communication systems, digital TVs, and multimedia systems, which has resulted in a demand for narrow transition-width FIR digital filters. A major drawback for the FIR filter is the large number of arithmetic operations needed for its implementation if the transition-width is narrow, which prevents it from providing a single chip solution for the most filtering applications. For years, considerable attention and efforts have been made on reducing the implementation complexities of FIR filters. One of the most efficient ways to design narrow transition-width arbitrary bandwidth FIR filter is the frequency-response masking technique [1- 7]. The basic idea behind this is to compose the overall filter using several subfilters, namely, the bandedge shaping filter, its complementary, and two masking filters. The bandedge shaping filter, $H_a(z^M)$, is derived by replacing each delay element of a prototype filter $H_a(z)$ by M delay elements. Its complementary filter $H_c(z^M)$ is given by

$$H_c(z^M) = z^{-M(N_a-1)/2} - H_a(z^M) \qquad (1)$$

where N_a is the filter length of $H_a(z)$. The frequency response of $H_a(e^{jM\omega})$ and $H_c(e^{jM\omega})$ are shown in Fig. 1(a). Two masking filters, $H_{Ma}(z)$ and $H_{Mc}(z)$, are used to remove the undesired frequency components from $H_a(z^M)$ and $H_c(z^M)$, respectively. The overall filter is formed according to

$$H(z) = H_a(z^M)H_{Ma}(z) + [z^{-M(N_a-1)/2} - H_a(z^M)]H_{Mc}(z) \qquad (2)$$

where $H(z)$ is z-transform transfer function of the overall filter. The length of $H_{Ma}(z)$ and $H_{Mc}(z)$ must be equal to ensure that the outputs from the two branches are in phase. If they are not of the same length, zero-valued coefficients must be padded to the shorter one. The frequency response of $H_{Ma}(z)$, $H_{Mc}(z)$ and $H(z)$ are shown in Fig. 1(b) and 1(c), and one of the implementation structures is shown in Fig.2.

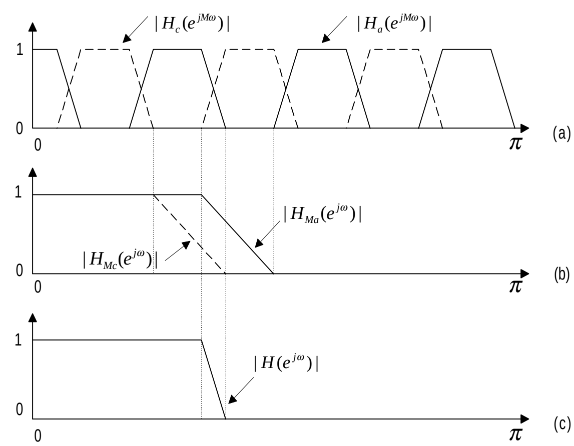

Fig. 1 The frequency response of subfilters used in the frequency-response masking approach

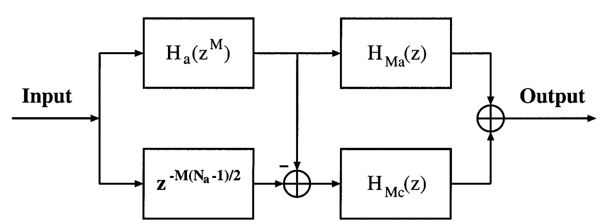

Fig.2 A realization structure of frequency-response masking approach

It is noted that the length of bandedge shaping filter, $H_a(z)$, can be quite long in a single stage frequency-response masking design. This is due to the fact that the transition-width Δ_a of the bandedge shaping filter $H_a(z)$ is determined by the product of the transition bandwidth Δ of the overall filter and the up-sampling rate M, i.e. $\Delta_a = M\Delta$. It is obvious that Δ_a will have a small value if Δ is very small, which leads to a long filter of $H_a(z)$. One proposed solution to address this problem is to implement $H_a(z)$ itself by using frequency response masking approach, which results in a multi-stage frequency-response masking implementation [2]. The drawback of multistage implementation is its high complexity associated with the design of each subfilters. In this paper we present a modified frequency-response masking approach to reduce the complexity of $H_a(z)$. In our new approach, $H_a(z)$ is designed using interpolated finite-impulse response (IFIR) technique [8]. The new approach is simple in structure and easy to apply. The overall filter can be synthesized by using existing algorithms, such as Remez and MILP.

2. A NEW STRUCTURE

When a narrow transition-width arbitrary bandwidth FIR filter is designed by using frequency response masking technique, it is always possible to find a suitable up-sampling rate M such that the bandedge shaping filter, $H_a(z)$, is a narrow band filter. When $H_a(z)$ is a narrow band filter, it can be implemented efficiently by Interpolated Finite-Impulse Response (IFIR) technique. The IFIR technique is based on the realization of the filter as a cascade of two subfilters. One of the filters has a sparse coefficient vector with periodic frequency response, and the other filter is cascaded to the first filter to attenuate the undesired frequency components. By replacing $H_a(z)$ with an IFIR filter, a modified frequency-response masking approach is formed. The *z*-transform transfer function of the overall filter can be written as

$$H(z) = H_s(z^{lM})H_g(z^M)H_{Ma}(z) + [z^{-(\frac{N_s-1}{2}l + \frac{N_g-1}{2})M} - H_s(z^{lM})H_g(z^M)]H_{Mc}(z) \qquad (3)$$

where $H_s(z^{lM})H_g(z^M)$ is an IFIR pair to replace $H_a(z^M)$. N_s and N_g are the filter length of $H_s(z)$ and $H_g(z)$, respectively. The frequency-response of various subfilters are shown in Fig.3. In Fig.3(a), $H_s(z)$ is a prototype bandedge shaping filter with bandedges at θ_s and ϕ_s, and its transition width is l times wider than that of the $H_a(z)$. Replacing each delay element of $H_s(z)$ by l delay elements, a filter $H_s(z^l)$ with frequency response $H_s^{'}(e^{j\omega})$ is obtained as shown in Fig. 3(b). Although the transition width of $H_s(z^l)$ is l times narrower that of the $H_s(z)$, it introduces some undesired passbands in the region ϕ_g to π. If a mask filter $H_g(z)$ with bandedges at θ_g and ϕ_g is used to remove the undesired passbands as shown in Fig.3(c), a $H_a(z)$ equivalent of $H_s(z^l)H_g(z)$ is formed as shown in Fig. 3(d). If the passband width of $H_{Ma}(z)$ is wider than that of $H_{Mc}(z)$, as shown in Fig. 3(e), we have

$$m = \left\lfloor \frac{\omega_p M}{2\pi} \right\rfloor \qquad (4a)$$

$$\theta_s = (\omega_p M - 2m\pi)l \qquad (4b)$$

$$\phi_s = (\omega_s M - 2m\pi)l \qquad (4c)$$

$$\theta_g = \omega_p M - 2m\pi \qquad (4d)$$

$$\phi_g = \frac{2\pi}{l} - (\omega_s M - 2m\pi) \qquad (4e)$$

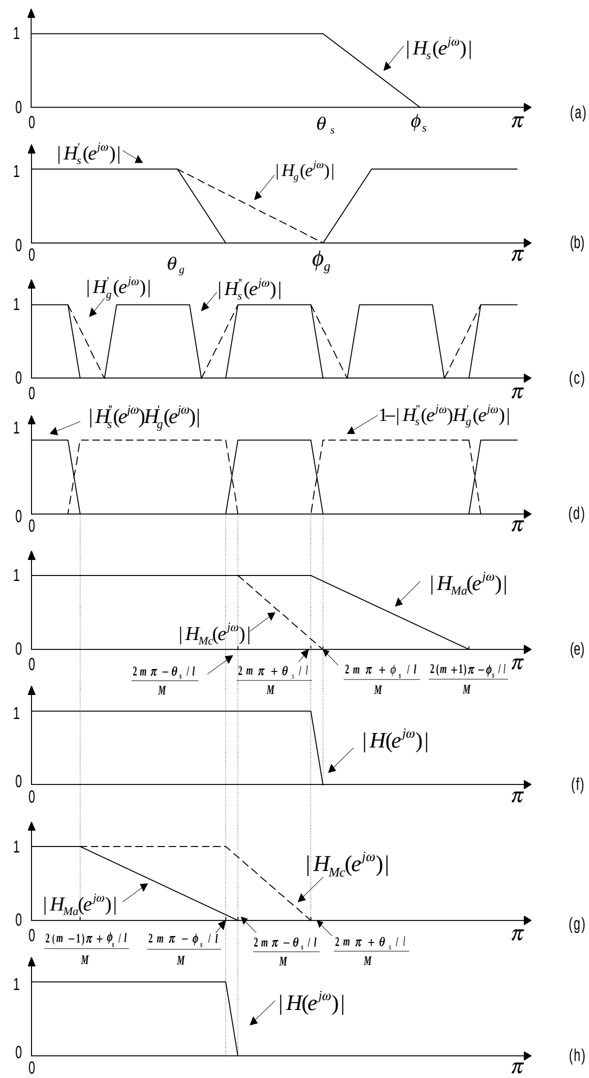

Fig. 3 The frequency-response of various subfilters in modified frequency-response masking approach

where $\lfloor x \rfloor$ denotes the largest integer less than x. ω_p and ω_s are the band edges of the overall filter $H(z)$. Fig. 3(f) shows the frequency response of $H(z)$. To ensure $\phi_s < \pi$, l must satisfy the condition

$$2 \le l \le \left\lfloor \frac{\pi}{\omega_s M - 2m\pi} \right\rfloor \qquad (4f)$$

If the passband width of $H_{Mc}(z)$ is wider than that of $H_{Ma}(z)$, as shown in Fig. 3(g), the overall frequency response $H(e^{j\omega})$ is shown in Fig .3(h). In this case, we have

$$m = \left\lceil \frac{\omega_s M}{2\pi} \right\rceil \qquad (5a)$$

$$\theta_s = (2m\pi - \omega_s M)l \qquad (5b)$$

$$\phi_s = (2m\pi - \omega_p M)l \qquad (5c)$$

$$\theta_g = 2m\pi - \omega_s M \qquad (5d)$$

$$\phi_g = \frac{2\pi}{l} - (2m\pi - \omega_p M) \qquad (5e)$$

$$2 \le l \le \left\lfloor \frac{\pi}{2m\pi - \omega_p M} \right\rfloor \qquad (5f)$$

where $\lceil x \rceil$ denotes the smallest integer larger than x. A realization structure of the overall filter for the modified frequency-response masking approach is shown in Fig. 4.

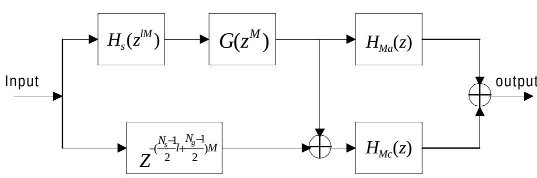

Fig.4 A realization structure of modified frequency-response masking approach

3. DESIGN PROCEDURES

It is important to examine the ripple effect of each subfilter $H_s(z^{lM})$, $H_g(z^M)$, $H_{Ma}(z)$ and $H_{Mc}(z)$ on the overall filter $H(z)$ before we embark on a design procedure. Using the frequency response to denote the overall filter, Eq.(3) can be simplified to

$$H(e^{j\omega}) = H_s^{''}(e^{j\omega})H_g^{'}(e^{j\omega})[H_{Ma}(e^{j\omega}) - H_{Mc}(e^{j\omega})] + H_{Mc}(e^{j\omega}) \qquad (6)$$

Let $D(\omega)$ and $\delta(\omega)$ denote the gain and deviation of the overall filter. $D_s^{''}(\omega)$ and $\delta_s^{''}(\omega)$ represent the gain and deviation of $H_s^{''}(e^{j\omega})$. Similar representations are applied to $H_g^{'}(e^{j\omega})$, $H_{Ma}(z)$ and $H_{Mc}(z)$, respectively. Let Case A represents the case as shown in Fig. 3(e) and Case B represents the case as shown in Fig. 3(g). We examine the ripple effect of each subfilter on the frequency response of the overall filter over the following 4 frequency ranges.

Frequency range 1: $0 \le \omega \le \frac{2m\pi - \theta_s / l}{M}$ in case A or $0 \le \omega \le \frac{2(m-1)\pi + \phi_s / l}{M}$ in case B. In this frequency range, $D(\omega) = D_{Ma}(\omega) = D_{Mc}(\omega) = 1$, Eq. (6) can be simplified to

$$\delta(\omega) = [D_s^{''}(\omega) + \delta_s^{''}(\omega)][D_g^{'}(\omega) + \delta_g^{'}(\omega)][\delta_{Ma}(\omega) - \delta_{Mc}(\omega)] + \delta_{Mc}(\omega) \qquad (7)$$

Ignoring the second order terms, we have

$$\delta(\omega) \approx \delta_{Ma}(\omega)\text{, for } D_s^{''}(\omega) = D_g^{'}(\omega) = 1 \qquad (8)$$

$$\delta(\omega) \approx \delta_{Mc}(\omega)\text{, for } D_s^{''}(\omega) \times D_g^{'}(\omega) = 0 \qquad (9)$$

Frequency range 2: $\frac{2m\pi - \theta_s / l}{M} < \omega < \frac{2m\pi + \theta_s / l}{M}$ in case A or $\frac{2(m-1)\pi + \phi_s / l}{M} < \omega < \frac{2m\pi - \phi_s / l}{M}$ in case B. In case A, $D(\omega) = D_{Ma}(\omega) = D_s^{''}(\omega) = D_g^{'}(\omega) = 1$. Neglecting the second terms, we have

$$\delta(\omega) = \delta_{Ma}(\omega) + [\delta_s^{''}(\omega) + \delta_g^{'}(\omega)][1 - D_{Mc}(\omega)] \qquad (10)$$

In this frequency range, $D_{Mc}(\omega)$ decreases from unity to zero as ω increases. Hence

$$|\delta(\omega)| \le |\delta_{Ma}(\omega)| + |\delta_s^{''}(\omega)| + |\delta_g^{'}(\omega)| \qquad (11)$$

In case B, $D(\omega) = D_{Mc}(\omega) = 1$, $D_s^{''}(\omega) \times D_g^{'}(\omega) = 0$. Neglecting the second term, Eq.(6) simplifies to

$$\delta(\omega) \approx [D_g^{'}(\omega)\delta_s^{''}(\omega) + D_s^{''}(\omega)\delta_g^{'}(\omega)][D_{Ma}(\omega) - 1] + \delta_{Mc}(\omega) \qquad (12)$$

$D_{Ma}(\omega)$ decreases from unity to zero. $D_s^{''}(\omega)$ and $D_g^{'}(\omega)$ are not equal to zero at the same frequency. Hence

$$|\delta(\omega)| \le \max\{|D_s^{''}(\omega)\delta_g^{'}(\omega)|, |D_g^{'}(\omega)\delta_s^{''}(\omega)|\} + |\delta_{Mc}(\omega)| \qquad (13)$$

Frequency range 3: $\frac{2m\pi + \phi_s / l}{M} < \omega < \frac{2(m+1) - \phi_s / l}{M}$ in case A or $\frac{2m\pi - \theta_s / l}{M} < \omega < \frac{2m\pi + \theta_s / l}{M}$ in case B. In case A, $D(\omega) = D_{Mc}(\omega) = 0, D_s^{''}(\omega) \times D_g^{''}(\omega) = 0$. Neglecting the second terms, we have

$$\delta(\omega) \approx D_{Ma}(\omega)[D_s^{'}(\omega)\delta_g^{'}(\omega) + D_g^{'}(\omega)\delta_s^{''}(\omega)] + \delta_{Mc}(\omega) \qquad (14)$$

$D_{Ma}(\omega)$ decreases from unity to zero. $D_s^{''}(\omega)$ and $D_g^{'}(\omega)$ are not equal to zero at the same frequency. Hence

$$|\delta| \le |\max\{|D_s^{''}(\omega)\delta_g^{'}(\omega)|, |D_g^{'}(\omega)\delta_s^{''}(\omega)|\} + |\delta_{Mc}(\omega)| \quad (15)$$

In case B, $D(\omega) = D_{Ma}(\omega) = 0$, $D_s^{''}(\omega) = D_g^{'}(\omega) = 1$. Neglecting the second terms, Eq.(6) simplifies to

$$\delta(\omega) \approx \delta_{Ma}(\omega) - D_{Mc}(\omega)[\delta_s^{''}(\omega) + \delta_g^{'}(\omega)] \quad (16)$$

In this frequency range, $D_{Mc}(\omega)$ decreases from unity to zero as ω increases. Hence

$$|\delta(\omega)| \le |\delta_{Ma}(\omega)| + |\delta_s^{''}(\omega)| + |\delta_g^{'}(\omega)| \quad (17)$$

Frequency range 4: $\frac{2(m+1)\pi - \phi_s/l}{M} \le \omega \le \pi$ in case A or $\frac{2m\pi + \theta_s/l}{M} \le \omega \le \pi$ in case B. In this frequency range, $D(\omega) = D_{Ma}(\omega) = D_{Mc}(\omega) = 0$, Eq.(6) simplifies to

$$\delta(\omega) = [D_s^{''}(\omega) + \delta_s^{''}(\omega)][D_g^{'}(\omega) + \delta_g^{'}(\omega)][\delta_{Ma}(\omega) - \delta_{Mc}(\omega)] + \delta_{Mc}(\omega) \quad (18)$$

Ignoring the second order terms.

$$\delta(\omega) \approx \delta_{Ma}(\omega), \text{ for } D_s^{''}(\omega) = D_g^{'}(\omega) = 1 \quad (19)$$

$$\delta(\omega) \approx \delta_{Mc}(\omega), \text{ for } D_s^{''}(\omega) \times D_g^{'}(\omega) = 0 \quad (20)$$

The above analysis provides us very useful information in synthesizing the filter using our modified frequency-response masking technique. It is obvious that in frequency ranges 1 and 4, the ripple of the overall filter is mainly determined by either $H_{Ma}(z)$ or $H_{Mc}(z)$. This property can be used to reduce the ripple constrains for both $H_{Ma}(z)$ and $H_{Mc}(z)$. It is also noted that the IFIR cascade pair only needs to satisfy the design specification within the frequency range 3 and 4. According to the above, the following design procedures are recommended to synthesis the filter:

Step 1. Choose M and l to satisfy Eq. (4f) for case A or Eq. (5f) for case B.

Step 2. Design of $H_{Ma}(z)$. The passband edge and stopband edge are shown in Fig. 3(e) for case A, and Fig. 3(g) for case B. In frequency range 1 and 4, the passband and stopband ripple constrains can be relaxed according to Eq. (9) and (20).

Step 3. Design of $H_{Mc}(z)$. The passband edge and stopband edge are shown in Fig. 3(e) for case A, and Fig. 3(g) for case B. In frequency range 1 and 4, the passband and stopband ripple constrains can be relaxed according to Eq. (8) and (19).

Step 4. Design of $H_g(z)$. The band edge θ_g and ϕ_g are given by Eq.(4d) and (4e) for case A and Eq.(5d) and (5e) for case B. According to the above discussion, we only need to design the filter in frequency ranges 2 and 3. For the case A, Eq. (11) and (15) must be satisfied. For the case B, Eq. (13) and (17) must be satisfied.

Step 5. Design of $H_s(z)$. Take $H_g(z)$ as the prefilter to design $H_s(z)$. The band edge θ_s and ϕ_s are given by Eq. (4b) and (4c) for case A and Eq. (5b) and (5c) for case B. In frequency ranges 2 and 3, it must satisfy Eq.(11) and (15) for the case A, and Eq(13) and (17) for the case B.

4. AN EXAMPLE

Now we shall illustrate our method by using an example. Consider the design of a linear phase FIR lowpass filter with passband edge at $\omega_p = 0.3 \times 2\pi$, stopband edge at $\omega_s = 0.301 \times 2\pi$. The maximum passband deviation is $0.1dB$ and the minimum stopband attenuation is $-80dB$. The estimated length of such a filter is 3177 by conventional design. If the original frequency response masking technique is used, the minimum length of the filter is estimated to be 427 for $M = 14$. The filter length of $H_a(z)$, $H_{Ma}(z)$ and $H_{Mc}(z)$ are 235, 80 and 112, respectively. 214 multipliers are needed. To design the filter using the proposed method, assume that the ripple magnitude of $H_s(z)$, $H_g(z)$, $H_{Ma}(z)$ and $H_{Mc}(z)$ are each 85% of the allowed ripple magnitude. For $M = 7$ and $l = 3$, the total filter length to meet the given specification is 326.

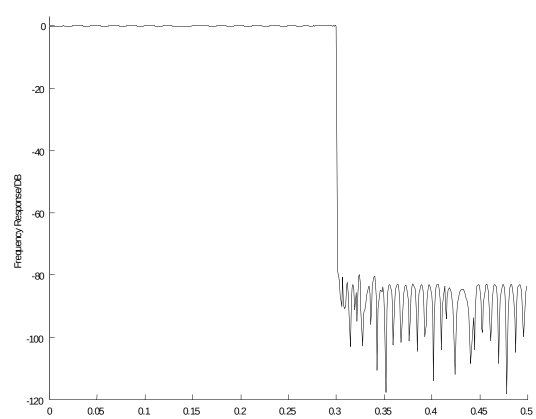

Fig.5 The frequency-response of the overall filter in Example

The length of $H_s(z)$, $H_g(z)$, $H_{Ma}(z)$ and $H_{Mc}(z)$ are 157, 27, 30 and 112, respectively. 164 multipliers are needed. This achieves about 24% savings in the number of multipliers compared to the original frequency-response masking approach. Fig. 5 shows the overall magnitude response.

5. CONCLUSIONS

A new structure to reduce the complexity of frequency response masking approach is proposed The design procedure for the proposed structure is derived. Our new method is simple and easy to apply compared to multistage frequency-response masking approach, and it can achieve considerably large savings in term of number of multipliers.

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