A HIGH-RESOLUTION TIME-VARYING SPECTRUM FOR CHIRP SIGNALS

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ABSTRACT

A high-resolution time-varying spectrum is proposed for the analysis of chirp signals. The time-varying spectrum is generated by combining an optimally weighted filter bank and the optimally weighted Wigner-Ville distributions (WDs) of the subband signals from the weighted filter bank. The weights for both the filter bank and the WDs are obtained by using least squares estimation. Simulation examples are given, with comparison in terms of performance measure, to show the efficiency of the new method.

1. INTRODUCTION

A chirp signal, which can be represented as sinusoid with time-varying amplitude and frequency, is commonly used in the communication applications. A high-resolution time-varying spectrum is necessary for the analysis of such non-stationary signals. A number of methods have been developed, specially for deterministic signal analysis; the most common method is the spectrogram, which is obtained as the squared modulus of the short-time Fourier transform (STFT) [1]. For stochastic processes, one of the early methods for the time-frequency analysis is the Wigner-Ville spectrum (WVS), defined by Martin and Flandrin as [2]

$$\mathbf{W}(t,f) = E\{\int x^*(t-\tau/2)x(t+\tau/2)e^{-j2\pi f\tau}d\tau\} \quad (1)$$

where the integral is the Wigner distribution (WD) of the process $x(t)$, $E\{.\}$ denotes the expectation operator and '$*$' stands for complex conjugate.

If several realizations of the nonstationary process $x(t)$ is available then one can obtain an estimate of the WVS by ensemble averaging the individual WD's of each realization. In case of single realization some smoothing of the WD of $x(t)$ is done for estimating the WVS [2]. Sometimes signal-dependent kernels can be applied to optimally smooth the WD and estimate the WVS [3, 4].

Recently, an alternative approach to WVS estimation was proposed in [5]. Thomson's multiple window stationary spectral estimation technique [6] is extended to the nonstationary case by averaging multiple Hermite windowed spectrograms of the de-chirped process $x(t)$. This method needs chirp extraction which is computationally quite demanding, and fails when two or more crossing chirps are present in the signal.

In this paper, we propose a high-resolution time-varying spectrum for the analysis of chirp signals. The method is based on the use of filter bank and the WDs which are optimally weighted. This is further development of the method presented in [7] for nonstationary WVS estimation. Examples are given, with performance criteria measures, to show the effectiveness of this new method.

2. HIGH-RESOLUTION TIME-VARYING SPECTRUM

Our aim is to generate a high resolution time-varying spectrum for chirp signals. We achieve this goal based on the combination of the following two steps: The first is to find a filter bank as optimally weighted using the least squares criteria. The second is to calculate the weighted average of WDs (for the subband signals of the optimally weighted filter bank) where the weights are determined with respect to the WD (for the input signal $x(n)$) in the weighted least square sense.

So, given $x(n), 0 \le n \le (l-1)$ we calculate M $(i = 1, \cdots, M)$ different WDs,

$$W_i(n,k) = \sum_{m=-(K-1)/2}^{(K-1)/2} w(m)z_i(n+m)z_i^*(n-m)e^{-j4\pi mk/K} \quad (2)$$

where $z_i(n)$ is the complex signal at the ith frequency band and n, $k(0 \le k \le K)$ and m are the discrete variables corresponding to time, frequency, time-lag whereas $w(m)$ is a real window. The $z_i(n)$ is obtained as

$$z_i(n) = x(n) \star h_i^{'}(n) \quad (3)$$

where '$\star$' denotes the convolution operator and $h_i(n)$ is the impulse responses of the subfilters of a uniform bandpass filter bank given by

$$h_i(n) = h(n)e^{j2\pi f_i n}, \quad i = 1, \cdots, M. \quad (4)$$

In (4), $h(n)$ is the real-valued impulse response of a prototype lowpass filter and M is the number of subfilters. Each subfilter is obtained by modulating the lowpass filter by a complex exponential with the normalized frequency $f_i = i/L, i = 1, \cdots, M$. The transfer functions of the subfilters are

$$H_i(f) = H(f - f_i) \quad i = 1, \cdots, M. \quad (5)$$

We have chosen $h(n)$ with the following conditions [8]

$$h(0) = 1/L, \quad h(lL) = 0 \quad \forall l \neq 0 \quad (6)$$

and put the following constraint on the filter bank in order to fulfill the time and frequency marginals (see [7] for more detail).

$$\sum_{i=0}^{M} |H(f - f_i)|^2 = 1 \qquad 0 \leq f \leq M/L. \quad (7)$$

Using the least squares criteria, the optimum weight vector $\mathbf{c} = [c_1 \ \cdots c_M]^T$ ($M \times 1$ size) of the filter bank can be obtained by solving the linear system [9]

$$\mathbf{Qc} = \mathbf{d} \quad (8)$$

In (8), $\mathbf{Q} = \sum_{n=0}^{l-1} \mathbf{h}(n)\mathbf{h}^T(n)$ is a Hermitian and positive definite matrix of size ($M \times M$) where $\mathbf{h}(n) = [h_1(n) \cdots h_M(n)]^T$ containing the impulse responses of the filters and l is the filter length. Also, in (8), $\mathbf{d} = \sum_{n=0}^{l-1} \mathbf{h}(n)f(n)$ is the M component column vector where $f(n)$ is the matched filter sequence which is the time reversal of the input sequence $x(n)$ of length l. Since the matrix Q is positive definite, (8) can be solved using the Cholesky factorization.

The impulse response of the optimally weighted filter bank will be then

$$h_i^{'}(n) = c_i h(n)e^{j2\pi f_i n}, i = 1, \cdots, M. \quad (9)$$

In the second step, we optimally weight each of the WDs, $W_i(n,k), i = 1, \cdots, M$, by solving the weighted least square (WLS) problem

$$\begin{aligned} \min_{\gamma_i} \sum_n \sum_k \quad & \psi(n,k)[R(n,k) - \textstyle\sum_{i=1}^{M} \gamma_i W_i(n,k)] \\ & [R(n,k) - \textstyle\sum_{i=1}^{M} \gamma_i W_i(n,k)]^* \end{aligned} \quad (10)$$

where γ_i's are the weights and $R(n,k)$ is a reference time-frequency distribution, which is the WD of the $x(n)$ in our case, $\psi(n,k)$ is a real and positive weighting function. For constant weighting $\psi(n,k)$=1.

Using a one-dimensional indexing (via row-wise scanning) of $W_i(n,k)$ and $R(n,k)$, we can write (10) in matrix form as

$$\min_{\gamma}[\mathbf{R} - \mathbf{W}\gamma]^* \Psi [\mathbf{R} - \mathbf{W}\gamma] \quad (11)$$

where $\gamma = [\gamma_1 \cdots \gamma_M]^T$is a column vector and Ψ is the weighting matrix (positive definite and Hermitian). For simplicity, we have chosen the weighting matrix Ψ to be equal to the identity matrix. From (11) the optimal weights, γ^{opt}, are then obtained as

$$\gamma^{\text{opt}} = (\mathbf{W}^*\Psi\mathbf{W})^{\dagger}\mathbf{W}^*\Psi\mathbf{R}. \quad (12)$$

In (12) $(\mathbf{W}^*\Psi\mathbf{W})^{\dagger}$ designates the pseudo inverse of the matrix $(\mathbf{W}^*\Psi\mathbf{W})$ ($M \times M$ size) that is easily calculated using the Cholesky factorization technique, since the above matrix is a symmetric positive-definite matrix.

Then the weighted least square estimate of $\hat{\mathbf{R}}_{ls}$ of $\mathbf{R}$ is

$$\hat{\mathbf{R}}_{ls} = \sum_{i=1}^{M} \gamma_i^{\text{opt}} W_i(n,k). \quad (13)$$

Since negative values have no physical interpretation in a time-varying spectrum, we consider the positive part of (13). Thus the non-linear filter bank based time-varying spectrum, we propose, is

$$\hat{\mathbf{R}} = \left\{ \sum_{i=1}^{M} \gamma_i^{\text{opt}} W_i(n,k) \right\}^+ \quad (14)$$

where $\{.\}^+$ denotes the positive part.

3. SIMULATION RESULTS

In this section, we have shown simulation results for noise-free and noisy signals. The simulated noisy signal is of the form

$$\begin{aligned} x(n) \quad &= s(n) + v(n) \\ &= Re\{\textstyle\sum_{j=1}^{P} a_j(n)e^{j\phi_j(n)}\} + v(n), \\ & n = 0, 1, \cdots, N-1 \end{aligned} \quad (15)$$

where

$$a_j(n) = b_j(\psi_j/\pi)^{1/4}e^{-\psi_j(n-n_{oj})^2/2} \quad (16)$$

and

$$\phi_j(n) = \alpha_j n^2/2 + \rho_j \cos(2\pi n/N) + \omega_{oj} n \quad (17)$$

are time-varying amplitude and phase, respectively of the jth signal component, $v(n)$ is the additive white Gaussian noise (AWGN) and $Re\{.\}$ denotes the real-part.

The signal-to-noise ratio(SNR) of a noisy signal in decibels (dB) is defined as

$$\text{SNR} = 10\log_{10}\left\{\frac{1/N\sum_{n=0}^{N-1}|s(n)|^2}{1/N\sum_{n=0}^{N-1}|v(n)|^2}\right\} \qquad (18)$$

We consider a multicomponent chirp signal that consists two chirp signals, i.e. a sinusoidal frequency modulated nonlinear chirp and a linear chirp. The respective parameters used in (16) and (17) are: $b_1 = 0.81$, $b_2 = 0.9$, $\psi_1 = 20/N^2$, $\psi_2 = 40/N^2$, $n_{01} = 128$, $n_{02} = 102$, $\alpha_1 = 0$, $\alpha_2 = (96/N^2)\pi$, $\rho_1 = 20$, $\rho_2 = 0$, $\omega_{01} = (60/N)\pi$, $\omega_{02} = (120/N)\pi$. The other parameters are N=256, K=128, and L=20. Fig. 1(a)–(b) show the WD and proposed time-varying spectrum for the noise-free case. Fig. 1(c)–(f) show the WD, proposed time-varying spectrum, spectrogram (using Hamming window), and Choi-Williams distribution (CWD) (using $\sigma = 0.5$ [11]) in the presence of zero-mean AWGN with SNR=3 dB. As seen from Fig. 1, the proposed method performs well in terms of cross-terms and noise reduction as well as high energy concentration.

4. PERFORMANCE MEASURE

In order to measure the performance we consider the statistically independent jth instantaneous frequencies, $\phi_j^{'}(t)$ (in radian), and the instantaneous bandwidth, $\sigma_j^2(t)$ of a multi-component signal.

Then the maximum entropy joint density (MEJD) can be given by [10]

$$\begin{aligned}\bar{P}_r(t,\omega) &= \bar{P}_r(t)\bar{P}_r(\omega|t)\\ &= \sum_{j=1}^{P}\frac{p_{rj}}{p_r}\frac{1}{\sqrt{2\pi\sigma_j^2(t)}}\bar{P}_{rj}(t)e^{-(\omega-\phi_j^{'}(t))^2/2\sigma_j^2(t)}\end{aligned} \qquad (19)$$

where $\bar{P}_r(\omega|t)$ is the conditional density, $\frac{p_{rj}}{p_r}$ is a normalization factor that is the ratio of the power of jth signal component to the total signal power over the time-frequency interval considered and $\bar{P}_{rj}(t)$is the time marginal of the jth signal component.

We use the true MEJD for an objective comparison between time-frequency representations (TFRs) for known signals. A performance measure (PM) of the resulting TFRs with respect to the above true one is given by

$$PM = \frac{\sum_{n=0}^{N-1}\sum_{k=0}^{K-1}(\bar{P}_r(n,k))^2}{\sum_{n=0}^{N-1}\sum_{k=0}^{K-1}|\bar{P}_r(n,k)-\tilde{P}_r(n,k)|^2} \qquad (20)$$

Method	noise-free case	noisy case
Proposed	1.38	1.14
CWD	0.81	0.76
Spectrogram	0.80	0.75
WD	0.74	0.71

Table 1: Results of PM (Fig. 1).

where $\bar{P}_r(n,k)$ is the discrete MEJD of the noise-free signal, and $\tilde{P}_r(n,k)$ is the TFR to be evaluated.

For the signal in (15) with discrete time n the $\phi_j^{'}(n) = \alpha_j n - \rho_j\sin(2\pi n/N)2\pi/N + \omega_{oj}$ and $\sigma_j^2(n) = \psi_j/2$ and $\frac{p_{rj}}{p_r} = b_j^2$ [10], from which the MEJD of the noise-free signal can be calculated as in (19). The corresponding true MEJD (using (19)) is shown in Fig. 1(g) and the PM results are given in Table 1. From Table 1 it is found that the proposed method is the most effective, since it provides the highest PM values in both noise-free and noisy situations. Note that for the PM calculation we have normalized the TFRs with unit norm and in the above example the parameter values used to calculate the TFRs for the noise-free case and noisy case, are the same.

5. CONCLUSIONS

In this paper, we have presented an efficient method of time-varying spectrum for the non-stationary chirp signals. The results are shown for the noise-free as well as noisy signals and compared with the WD, spectrogram, and other time-frequency representations (e.g CWD). It is found that presented method outperformed the other time-vaying spectral analysis methods in terms of high resolution. For the proposed method the choice of the parameter L should not be very small (e.g. not less than 16) in order to reduce the cross-terms. The presented method could be useful for the analysis of other non-stationary signals, such as speech and audio signals. In this case a suitable filter bank could be used.

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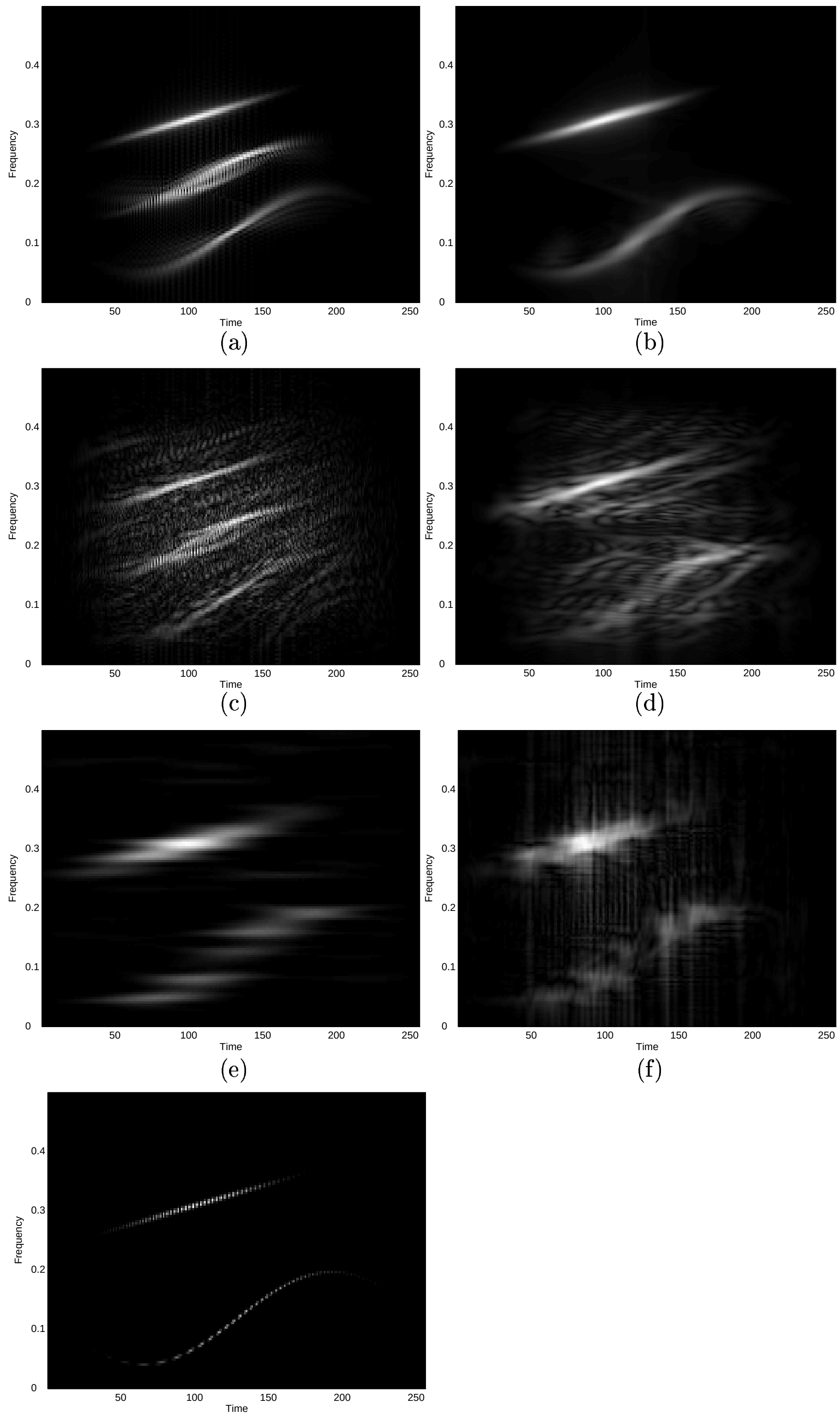

Figure 1: Results for the simulated (multicomponent) signal without noise ((a)–(b)) and with noise ((c)–(f)) case; (a) WD, (b) Proposed time-varying spectrum, (c) WD, (d) Proposed time-varying spectrum, (e) Spectrogram, (f) CWD, (g) True MEJD (using (19)).