A SUBBAND BLOCK ADAPTIVE ALGORITHM

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ABSTRACT

A subband block adaptive algorithm which has a quadratic cost function as with the LMS is proposed. The quadratic cost function is obtained through block process to the each polyphase components of the decomposed input signal by filter banks. This procedure gives the coefficient update equation as same with the block LMS algorithm. And a variable adaptive gain algorithm which use the same cost function is applied to the algorithm for increasing the convergence rate. The algorithm with variable adaptive gain has good convergence speed and small steady state error. Simulations prove the performance of the new algorithm.

1. INTRODUCTION

In gradient type adaptive algorithm, the representative is LMS, convergence speed improvement has been a major research theme during a decade. And it is well known that its convergence speed has close related to the adaptive gain and the computational complexity[1][2].

There are two techniques for improving convergence speed, time domain and frequency domain approaches. In time domain case, many algorithms with variable adaptive gain have been proposed, an important member is normalized LMS algorithm[1], and other methods used, for example, an optimum step size block of data or a fuzzy based adaptive gain method[3][4].

Generally, frequency domain adaptive algorithms, which update weight vector in frequency domain and have similar recursions of the block LMS algorithm are usually advantageous in computational complexity[2]. Specially, the subband adaptive technique which is regard as frequency domain method even though it may not utilize the FFT brings down the eigenvalue spread of the input signal correlation matrix of each band[5]. Both of the frequency domain methods generate approximately uncorrelated signals using the FFT or the filter banks. Therefore gradient adaptive filters that are realizes in the frequency domain have good convergence behaviors.

In subband adaptive filtering, the computational complexity can be reduced by polyphase decomposed adaptive filter structure[5][6]. In the case, the convergence speed is increases as increase the number of bands in the filter. However, if the cost function is linear combination of squared errors of each bands, the cost function is not quadratic form of filter coefficients of each bands. So, the convergence analysis is performed asymptotically[5].

In this paper, we proposed a new block subband adaptive algorithm. This method use the block input of each polyphase decomposed adaptive filters for a quadratic cost function. And we apply optimum variable adaptive gain to the algorithm for improving the convergence speed when the LMS is used as adaptive algorithm. The PR(Perfect Reconstruction) filter banks are used to avoid any distortion in analysis and synthesis[6]. In our presentation, we show the performances of the suggested algorithm by comparing simulation results with the conventional subband adaptive filter in[5].

2. BLOCK SUBBAND ADAPTIVE FILTERING

The subband adaptive filter structure using polyphase decomposition and noble identities is shown in Fig. 1[5]. Here, $S(z)$ is the unknown system with length $2L$, and $\hat{S}_0(z)$ and $\hat{S}_1(z)$ are polyphase components of the estimated unknown system. $H_0(z)$ and $H_1(z)$ are the analysis filters. The subband components of the input $x[n]$ are obtained by decimation from the outputs $b_0[n]$ and $b_1[n]$ of the filters $H_0(z)$ and $H_1(z)$. The error signals $e_0[n]$ and $e_1[n]$ are used to update the coefficients of the filters $\hat{S}_0(z)$ and $\hat{S}_1(z)$ through adaptive algorithm. Generally, in gradient adaptive algorithm, a quadratic cost function is desirable for the algorithm surely converges to global solution.

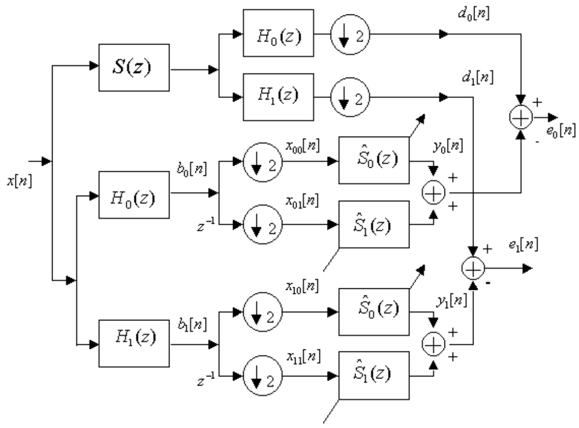

Figure 1: The subband adaptive filter structure

In the structure of Fig. 1, to obtain the instantaneous gradient like as the LMS, a cost function can be obtained as linear combination of squared errors ${e_0}^2[n]$ and ${e_1}^2[n]$, but it is not quadratic function of the coefficients of the filters $\hat{S}_0(z)$ and $\hat{S}_1(z)$[5]. To obtain a quadratic cost function, we process the input signals $x_{00}[n]$, $x_{01}[n]$, $x_{10}[n]$ and $x_{11}[n]$ to L-point blocks respectively. The $L \times L$ block(matrix) of $x_{00}[n]$ is $\mathbf{X}_{00}[n]$.

$$\mathbf{X}_{00}[n] = \begin{bmatrix} x_{00}[n] & \cdots & x_{00}[n-L+1] \\ x_{00}[n+1] & \cdots & x_{00}[n-L+2] \\ \vdots & \ddots & \vdots \\ x_{00}[n+L-1] & \cdots & x_{00}[n] \end{bmatrix} \tag{1}$$

As with the same method, we can get $\mathbf{X}_{01}[n]$, $\mathbf{X}_{10}[n]$ and $\mathbf{X}_{11}[n]$ corresponds to $x_{01}[n]$, $x_{10}[n]$ and $x_{11}[n]$, respectively. All the blocks of the input signal components make the $2L \times 2L$ matrix $\mathbf{X}[n]$ as follows.

$$\mathbf{X}[n] = \begin{bmatrix} \mathbf{X}_{00}[n] & \mathbf{X}_{01}[n] \\ \mathbf{X}_{10}[n] & \mathbf{X}_{11}[n] \end{bmatrix} \tag{2}$$

And the $L \times 1$ vectors of $y_0[n]$ and $y_1[n]$ are given by

$$\mathbf{y}_0[n] = [y_0[n], y_0[n+1], \cdots, y_0[n+L-1]]^T \tag{3}$$

$$\mathbf{y}_1[n] = [y_1[n], y_1[n+1], \cdots, y_1[n+L-1]]^T \tag{4}$$

,where T denotes transpose. Similarly, we obtain $\mathbf{d}_0[n]$, $\mathbf{d}_1[n]$, $\mathbf{e}_0[n]$ and $\mathbf{e}_1[n]$ of $d_0[n]$, $d_1[n]$, $e_0[n]$ and $e_1[n]$, respectively. From the equation (2), (3) and (4), we have outputs

$$\begin{bmatrix} \mathbf{y}_0[n] \\ \mathbf{y}_1[n] \end{bmatrix} = \begin{bmatrix} \mathbf{X}_{00}[n] & \mathbf{X}_{01}[n] \\ \mathbf{X}_{10}[n] & \mathbf{X}_{11}[n] \end{bmatrix} \begin{bmatrix} \hat{\mathbf{s}}_0[n] \\ \hat{\mathbf{s}}_1[n] \end{bmatrix} \tag{5}$$

and can be expressed simply by

$$\mathbf{X}_n \hat{\mathbf{S}}_n = \hat{\mathbf{Y}}_n. \tag{6}$$

And the errors are given by

$$\mathbf{E}_n = \mathbf{D}_n - \mathbf{Y}_n = \begin{bmatrix} \mathbf{d}_0[n] \\ \mathbf{d}_1[n] \end{bmatrix} - \begin{bmatrix} \mathbf{y}_0[n] \\ \mathbf{y}_1[n] \end{bmatrix}. \tag{7}$$

Then, according to [7], the cost function of this case is defined as

$$\mathbf{\Psi}_n \equiv \frac{1}{2L} E[{\mathbf{E}_n}^T \mathbf{E}_n] \tag{8}$$

where $E[\cdot]$ denotes the expectation operator. The instantaneous estimated cost function is given by

$$\hat{\mathbf{\Psi}}_n(\hat{\mathbf{S}}_n) = \frac{1}{2L}([{\mathbf{D}_n}^T \mathbf{D}_n] - 2[{\mathbf{D}_n}^T \mathbf{X}_n]\hat{\mathbf{S}}_n + \hat{\mathbf{S}}_n^T [{\mathbf{X}_n}^T \mathbf{X}_n]\hat{\mathbf{S}}_n) \tag{9}$$

as similar with the LMS. From equation (9), we know that the instantaneous cost function is quadratic function of $\hat{\mathbf{s}}_n$. The negative partial derivative of equation (9) with respect to $\hat{\mathbf{s}}_n$ is a estimated gradient vector $\mathbf{p}_n$ which is given by

$$\mathbf{p}_n = -\nabla \hat{\mathbf{\Psi}}_n(\hat{\mathbf{S}}_n) = \frac{1}{L} {\mathbf{X}_n}^T \mathbf{E}_n. \tag{10}$$

Therefore, the adaptation equation of the block subband filter can be expressed as

$$\hat{\mathbf{S}}_{n+1} = \hat{\mathbf{S}}_n + \frac{\mu}{L} {\mathbf{X}_n}^T \mathbf{E}_n \tag{11}$$

where μ is a adaptive gain. The adaptive gain must satisfy the condition, for the stability of the algorithm, $0 < \mu < \frac{1}{\lambda_{max}}$, where λ_{max} is the largest eigenvalue of ${\mathbf{X}^T}_n \mathbf{X}_n$[1][5].

3. VARIABLE ADAPTIVE GAIN FOR THE SUBBAND BLOCK ADAPTIVE FILTERING

Generally, the adaptive algorithm is more preferable to have a variable adaptive gain than have fixed one for a fast convergence[1][8]. The update equation with a variable adaptive gain can be given by

$$\hat{\mathbf{S}}_{n+1} = \hat{\mathbf{S}}_n + \mu_n \mathbf{p}_n \tag{12}$$

where $\mathbf{p}_n$ is the estimated gradient in equation (10). For the variable adaptive gain to be optimum at each iteration, it is need to select a proper function which have to be minimized with respect to the adaptive gain. With the similar method in [8], a function for the optimum adaptive gain is derived by substituting equation (12) to (9).

$$
\begin{aligned}
\hat{\boldsymbol{\Psi}}_n(\hat{\mathbf{S}}_{n+1}) = & \quad \hat{\boldsymbol{\Psi}}_n(\hat{\mathbf{S}}_n + \mu_n \mathbf{p}_n) \\
= & \quad \hat{\boldsymbol{\Psi}}_n(\hat{\mathbf{S}}_n) - \mu_n \mathbf{p}_n^T \mathbf{p}_n + \tfrac{\mu_n^2}{2L} \mathbf{p}_n^T [\mathbf{X}_n^T \mathbf{X}_n] \mathbf{p}_n.
\end{aligned} \tag{13}
$$

From equation (13), we know that $\hat{\boldsymbol{\Psi}}(\hat{\mathbf{s}}_{n+1}) - \hat{\boldsymbol{\Psi}}(\hat{\mathbf{s}}_n)$ is quadratic of μ_n. Therefore, the optimum adaptive gain at every iteration is obtained by differentiating the quadratic function with respect to μ_n and set to zero.

$$
\mu_n^{opt} = L \frac{\mathbf{p}_n^T \mathbf{p}_n}{\mathbf{p}_n^T \mathbf{X}_n^T \mathbf{X}_n \mathbf{p}_n}. \tag{14}
$$

It will be shown in the next section that the amount of additional computation for optimal μ_n does not exceed the convergence speed improvement due to the optimization.

4. SIMULATION RESULTS

In this section, we prove the good convergence properties of the subband block adaptive algorithm with fixed and variable adaptive gain. The performances are evaluated on the normalized coefficient error vector norm and the squared error in decibels respectively. The normalized coefficient error vector nom was defined in [5] as follows.

$$
10 \log_{10} \frac{\mathbf{v}_n^T \mathbf{v}_n}{\mathbf{S}_n^T \mathbf{S}_n} \tag{15}
$$

, where $\mathbf{v}_n$ is the difference with the unkown system coefficient vector $\mathbf{S}$ and adaptive filter coefficient vector $\hat{\mathbf{S}}_n$. In fixed gain case, the simulation results are compared with the fullband adaptive algorithm(NLMS) and subband adaptive filter of two bands in [5]. And the performances of the variable gain algorithm are compared with the fixed gain adaptive filter of this paper. The simulation conditions are set similarly with [5] for the performance comparison. The input signal is modeled as first-order autoregressive process with white Gaussian noise $w[n]$,

$$
x[n] = \rho x[n-1] + w[n] \tag{16}
$$

In simulations, ρ is fixed at 0.8., the length of unknown system is 8. And we added -30 dB system noise, the analysis filters are the Daubechies filter of order 16[9]. Simulation results of the fixed gain algorithm about the coefficient error vector norm and squared error are shown in Fig. 2. From this plots, we know that the convergence speed of the proposed algorithm is better than that of the other algorithms.
In Fig. 3, the norm and squared error are plotted for the variable gain algorithm. Note that the speed of

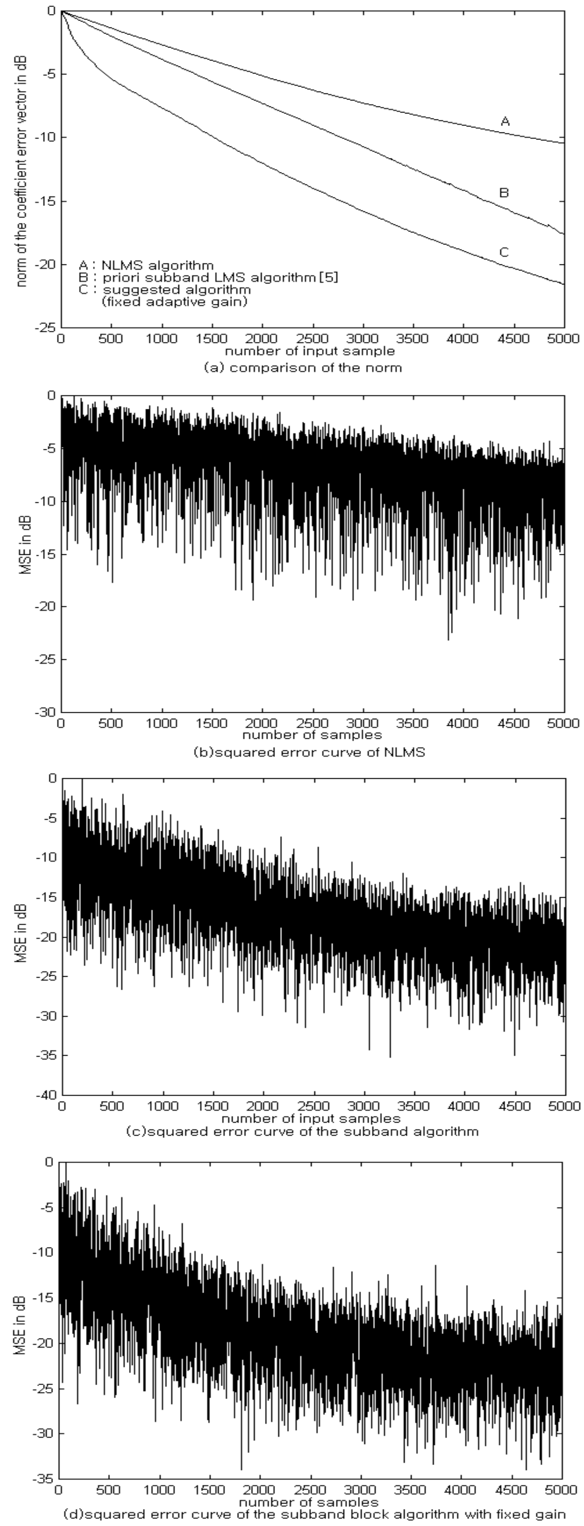

Figure 2: Comparison of the performance of the subband block algorithm, the subband algorithm, and NLMS(adaptive gain are 0.0051, 0.0043, and 0.54 respectively)

convergence is improved considerably. From the comparison of the Fig. 2(d) and Fig. 3(b), we know that the steady state behavior is improved by the variable adaptive gain. These experimental results explain the fast convergence and small steady state error associated with the algorithm.

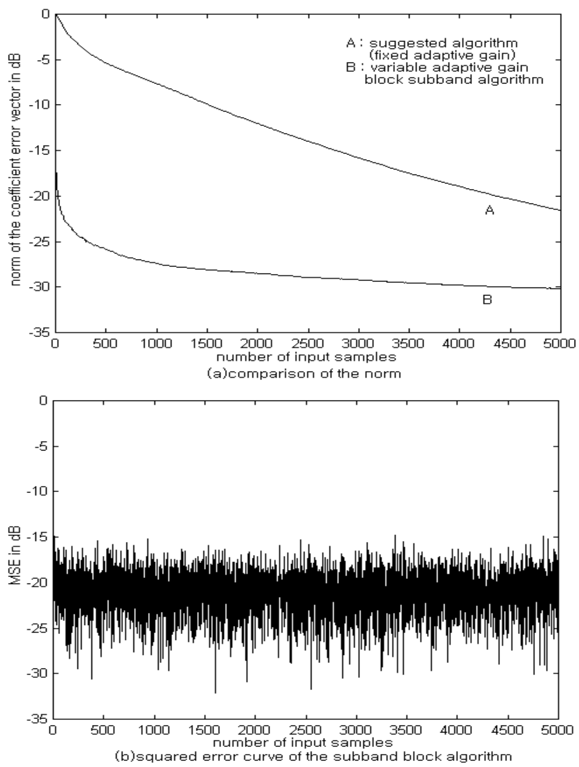

Figure 3: Performance comparison of the subband block algorithm with fixed and variable adaptive gain(fixed gain and initial value of the variable gain are same value of 0.0051)

5. CONCLUSIONS

This paper presented a subband block adaptive algorithm. The algorithm use a quadratic cost function to update filter coefficients. The cost function was obtained by using the $2L \times 2L$ input matrix which is composed of block matrix of the polyphase input components. The variable adaptive gain algorithm was derived, and applied to the algorithm for improving the convergence speed. The experimental results showed good convergence performances which is due to the orthogonality of the input signals decomposed by filter bank and the accurate estimation of the stochastic gradient by block procedures. And, the small steady state error resulted from the variable adaptive gain was measured also. The good properties of the algorithm will be improved by increasing the band of filter banks.

REFERENCES

[1] S. Haykin, *Adaptive Filter theory*, 2nd Ed. Upper Saddle River, N.J., Prentice-Hall, 1996.

[2] J. Shynk, "Frequency-domain and multirate adaptive filtering," *IEEE Signal Processing Magazine*, Vo1. 9, No. 1, pp. 14-37, Jan. 1992.

[3] G. Keratiotis and L. Lind, "Optimum variable step-size sequence for LMS adaptive filters", *IEE Proceeding Vision, Image and Signal Processing*, Vol. 146, No. 1, pp. 1-6, Feb. 1999.

[4] G. T. Ryu, D. S. Kim, D. Y. Lee, S. H. Han and H. D. Bae, "Convergence improvement of adaptive lattice algorithm with fuzzy based adaptive gain", *Journal of Circuits, Systems, and Computers*, Vol. 9, No. 1, pp. 125-132, Dec. 1999.

[5] S. S. Pradhan and V. U. Reddy, "A new approach to subband adaptive filtering", *IEEE Trans. on Signal Processing*, Vol. 47, No. 3, pp. 655-664, Mar. 1999.

[6] P. P. Vaidyanathan, "Multirate digital filters, filter banks, polyphase networks, and applications : a tutorial", *Proceedings of the IEEE*, Vol. 78, No. 1, pp. 56-93, Jan. 1990.

[7] G. A. Clark, S. R. Parker and S. K. Mitra, " Block implementation of adaptive digital filters," *IEEE Trans. on Circuits and Systems*, Vo1. CAS-28, No. 6, pp. 584-592, June 1981.

[8] J. S. Lim and C. K. Un, " Optimum block adaptive filtering algorithms using the preconditioning technique," *IEEE Trans. on Signal Processing*, Vo1. 45, No. 3, pp. 773-778, Mar. 1997.

[9] I. Daubechies, *Ten Lectures on Wavelets*, Philadelphia, Pennsylvania, SIAM, 1992.