

# FIR INVERSE FILTERS: MIMO SYSTEMS IN SEVERAL VARIABLES

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## ABSTRACT

A necessary and sufficient condition is given for the existence of a polynomial left inverse for a polynomial (FIR) system. Additionally, random systems are defined, and a sufficient condition is given for almost sure existence of a polynomial inverse. The two results extend previous work in single-input, multiple-output systems to the case of multiple input systems and to functions of several variables. Corollaries describe calibration of polarimetric, wide-band radar imagery.

## 1. INTRODUCTION

We consider finite impulse response (FIR) inverse filterbanks for the inversion of FIR distortion filterbanks. Such systems arise naturally in channel equalization for wireless communication systems with multiple antennas [15], in space object recognition [5] and in polarimetric calibration of radars [6, 17]. These inverse problems have three features in common. First, the systems have multiple outputs. Second, the measurement operator from any one input signal to any one output signal, with the suppression of the other inputs, if any, can be modeled as linear and shift-invariant (LSI). Each of the outputs can be modeled as a superposition of responses from each of the inputs. Third, the distortion on any data point is constrained to be in a finite neighborhood of that point. Thus, the distortion can be modeled as a bank of finite impulse response filters.

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## 2. SYSTEM MODEL

The data and the channel responses may be represented as polynomials in the transform domain  $k[z_1, \dots, z_t]$  where  $t$  is the dimension of the input data (the FIR convolution takes place in  $t$ -space). For our purposes, the  $z$ -transform of a data point  $x$  may be defined as

$$X(z) = \mathcal{Z}(x) = \sum_{\lambda \in \mathbb{Z}_{\geq 0}^t} x(\lambda) z^\lambda \quad (1)$$

where  $z = (z_1, \dots, z_t)$ ,  $\lambda = (\lambda_1, \dots, \lambda_t)$  is a  $t$ -tuple of nonnegative integers, and  $z^\lambda = z_1^{\lambda_1} z_2^{\lambda_2} \dots z_t^{\lambda_t}$ . Note that the sum above is always finite as all signals we consider have finite extent. Then, the equalization problem can be rephrased as follows.

Let the input polynomials be  $X_i(z_1, z_2, \dots, z_t)$ ,  $1 \leq i \leq m$ . We will suppress the independent variables  $z_1, z_2, \dots, z_t$ . Let the output polynomials  $Y_j$  be defined as

$$Y_j = \sum_{i=1}^m H_{ij} X_i \quad 1 \leq j \leq n \quad (2)$$

where  $H_{ij}$  are polynomials, too. Now, we seek *polynomials*  $G_{ij}$  such that

$$\sum_{j=1}^n G_{ij} Y_j = X_i \quad 1 \leq i \leq m \quad (3)$$

$$\Leftrightarrow \sum_{j=1}^n \sum_{k=1}^m G_{ij} H_{kj} X_k = X_i \quad 1 \leq i \leq m \quad (4)$$

In matrix notation (notice the nonstandard indexing of the elements of  $\mathcal{H}$ ), the preceding equations become

$$\mathcal{H} = \begin{pmatrix} H_{11} & H_{21} & \dots & H_{m1} \\ H_{12} & H_{22} & \dots & H_{m2} \\ \vdots & \vdots & \ddots & \vdots \\ H_{1n} & H_{2n} & \dots & H_{mn} \end{pmatrix} \quad (5)$$

$$\mathcal{G} = \begin{pmatrix} G_{11} & G_{12} & \cdots & G_{1n} \\ G_{21} & G_{22} & \cdots & G_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ G_{m1} & G_{m2} & \cdots & G_{mn} \end{pmatrix} \quad (6)$$

$$\mathcal{G}\mathcal{H} = I_m \quad (7)$$

where  $I_m$  denotes the  $m \times m$  identity matrix. Since constants are polynomials too, it is immediate from linear algebra that for solvability of the preceding equation, it is necessary that  $n \geq m$ .

### 3. EXISTENCE

#### 3.1. Necessary and Sufficient Conditions

**Theorem 1** *A matrix  $\mathcal{H}$  of size  $n \times m$ , whose elements are polynomials with coefficients in a complete field, has a polynomial left inverse  $\mathcal{G}$  iff*

1.  $n \geq m$  and
2. the ideal of  $m \times m$  minors of  $\mathcal{H}$  generates the ring of polynomials

*i.e., there is no zero common to all the  $m \times m$  minors of  $\mathcal{H}$ .*

**Proof.** Let  $\mathcal{H}_i$  denote the  $m \times m$  submatrices of  $\mathcal{H}$ . Let  $D_i$  denote the determinant of  $\mathcal{H}_i$ . If the ideal of  $m \times m$  minors of  $\mathcal{H}$  generates the ring of polynomials, there exist polynomials  $a_i$  such that

$$\sum_{i=1}^N a_i D_i = 1 \quad (8)$$

$$N = \binom{n}{m} \quad (9)$$

Then, for those  $\mathcal{H}_i$  such that  $D_i \neq 0$ , by Cramer's rule, there exist square polynomial matrices  $\mathcal{G}_i$  (adjoints of  $\mathcal{H}_i$ ) such that

$$\mathcal{G}_i \mathcal{H}_i = D_i I_m \quad (10)$$

Thus, there exist  $m \times n$  matrices  $\tilde{\mathcal{G}}_i$  which are  $\mathcal{G}_i$  (with additional zero rows corresponding to those columns in  $\mathcal{H}$  which are not in  $\mathcal{H}_i$ ) such that

$$\tilde{\mathcal{G}}_i \mathcal{H} = D_i I_m \quad (11)$$

from which it follows that

$$\sum_{i=1}^N (a_i \tilde{\mathcal{G}}_i) \mathcal{H} = \sum_{i=1}^N a_i D_i I_m = I_m \quad (12)$$

whence

$$\mathcal{G} = \sum_{i=1}^N (a_i \tilde{\mathcal{G}}_i) \quad (13)$$

Conversely, suppose that the  $m \times m$  minors of  $\mathcal{H}$  do not generate the ring of polynomials. Then, by the weak *Nullstellensatz* [4, p. 169],

$$\exists w = (w_1, w_2, \dots, w_t) \in k^t$$

such that  $D_i(w) = 0, 1 \leq i \leq N$ . Then  $\text{rank}(\mathcal{H}(w)) < m$ . Then, there does not exist  $\mathcal{G}(w)$  such that  $\mathcal{G}(w)\mathcal{H}(w) = I_m$ . Thus, a polynomial left inverse of  $\mathcal{H}$  does not exist. ■

For the single-input, multiple-output (SIMO) case in one variable, (*i.e.*,  $t = 1, m = 1, n > 1$ ), Theorem 1 is the classical Bezout equation [1] for which solution techniques are well known [1, 13, 14]. For the multiple-input, multiple-output (MIMO) case, Theorem 1 reduces the existence of a polynomial inverse to the SIMO case. The constructive proof requires solution of equation (8); therefore, the MIMO problem with  $mn$  filters  $H_{ij}$  is reduced to a SIMO problem with  $\binom{n}{m}$  filters  $D_i$ .

For the square case of  $m = n$ , Theorem 1 states that an FIR equalizer exists iff  $\det \mathcal{H}$  is a nonzero constant.

#### 3.2. Computing the Inverse

The constructive proof of Theorem 1 provides means for computing the FIR inverse in the MIMO case. First, a result of Berenstein and Yger [1] provides a tight upper bound on the required filter order for the FIR inverse. Let

$$\psi = \max_{1 \leq i \leq N} \deg(D_i) \quad (14)$$

**Proposition 2** *If  $D_i$  do not share a common zero, then there exist polynomials  $a_i$  of order  $2(2\psi)^{t-1}$  such that equation (8) is satisfied ([1], [9, Theorem 5] when  $t = 2$ ).*

Note that, for a given  $\psi$ , there exist  $D_i$  such that the bounds on the  $a_i$  are achieved. However, for a given  $D_i$ , there may exist lower order solutions  $a_i$  which satisfy equation (8). By reducing the problem of inversion of  $\mathcal{H}$  to the solving of equation (8), we have reduced the MIMO problem with filters  $H_{ij}$  to a SIMO problem with filters  $D_i$ .

Now, after zero padding the  $D_i$  so that they all have order  $\psi$ , the following linear equations yield the  $a_i$ :

$$\begin{bmatrix} \mathbf{D}_1 & \mathbf{D}_2 & \dots & \mathbf{D}_N \end{bmatrix} \begin{bmatrix} \mathbf{A}_1 \\ \mathbf{A}_2 \\ \dots \\ \mathbf{A}_N \end{bmatrix} = \begin{bmatrix} 1 \\ \mathbf{0} \end{bmatrix} \quad (15)$$

where  $\mathbf{D}_i$  is the convolution matrix determined by  $D_i$ ,  $1 \leq i \leq N$  and  $\mathbf{A}_i$  are the vectorized coefficients of  $a_i$ . In [16], this same approach is used when  $t = 1$ ,  $m = 1$  resulting in Sylvester matrices.

If the equalizer is not restricted to be FIR, then the problem can be solved using techniques analogous to the Smith-McMillan form and the matrix fraction description [13, chapters 2 and 6].

**Proposition 3** *If there exists a polynomial left inverse  $\mathcal{G}$  whose elements have (possibly) complex coefficients, for an  $\mathcal{H}$  whose elements have purely real coefficients, then there exists a left inverse  $\hat{\mathcal{G}}$  whose elements have purely real coefficients. Moreover, given  $\mathcal{G}$ ,*

$$\hat{\mathcal{G}} = \frac{(\mathcal{G} + \bar{\mathcal{G}})}{2}$$

is one such left inverse, where  $\bar{\mathcal{G}}$  denotes the matrix obtained by conjugating the coefficients of each polynomial in  $\mathcal{G}$ .

**Proof.** Polynomials are entire functions. Hence, for  $H_{kj}$  on the real manifold  $\mathbb{R}^t$ , which is a domain of uniqueness,

$$\sum_{j=1}^n G_{ij} H_{kj} = \delta_{ik} \quad 1 \leq i, k \leq m \quad (16)$$

$$\sum_{j=1}^n \bar{G}_{ij} H_{kj} = \delta_{ik} \Rightarrow \sum_{j=1}^n \frac{(G_{ij} + \bar{G}_{ij})}{2} H_{kj} = \delta_{ik} \quad (17)$$

Note that

$$\frac{(G_{ij} + \bar{G}_{ij})}{2}$$

is a holomorphic function as it is a polynomial. The result in equation (17) on  $\mathbb{C}^t$  then follows from the uniqueness theorem [11] for holomorphic functions. ■

### 3.3. In the Original Domain

Consider the problem of finding deconvolution filters in the original domain. Equation (4) on the desired equalizer can be recast as convolution in the original domain as follows:

$$\begin{aligned} x_i &= \sum_{j=1}^n g_{ij} \otimes \left( \sum_{k=1}^m h_{kj} \otimes x_k \right) & 1 \leq i \leq m \\ &= \sum_{k=1}^m \left( \sum_{j=1}^n g_{ij} \otimes h_{kj} \right) \otimes x_k & 1 \leq i \leq m \end{aligned} \quad (18)$$

**Lemma 4** *The equations (18) hold for arbitrary sets of  $x_i$ ,  $1 \leq i \leq m$  iff*

$$\sum_{j=1}^n g_{ij} \otimes h_{kj} = \delta_{ik} \quad 1 \leq i \leq m, \quad 1 \leq k \leq m \quad (19)$$

where  $\delta_{ik}$  is the original domain equivalent of the Kronecker delta function in the transform domain.

**Remark.** A constant  $C$  in the transform domain corresponds in the original domain to a function on  $\mathbb{Z}_{\geq 0}^t$  which is  $C$  at  $(0, 0, \dots, 0)$  and is zero elsewhere.

Now, given  $i$ , the filters  $g_{ij}$ ,  $1 \leq j \leq n$  are completely determined by the  $m$  equations (19). This decouples the problem of finding the inverse filters into  $m$  subproblems.

Filtering is a linear operation. Given a bound on the orders of the inverse filters  $(\pi_1, \dots, \pi_t)$ , and noting that all the filters involved are FIR filters, each convolution can be expressed as a matrix operation. Solving equation (19) for a given  $i$  is then equivalent to solving the following linear system of equations:

$$\underbrace{\begin{bmatrix} \mathbf{H}_{11} & \mathbf{H}_{12} & \dots & \mathbf{H}_{1n} \\ \mathbf{H}_{21} & \mathbf{H}_{22} & \dots & \mathbf{H}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{H}_{m1} & \mathbf{H}_{m2} & \dots & \mathbf{H}_{mn} \end{bmatrix}}_{\mathcal{F}} \begin{bmatrix} \mathbf{G}_{i1} \\ \mathbf{G}_{i2} \\ \vdots \\ \mathbf{G}_{in} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \vdots \\ \mathbf{L} \\ \vdots \\ \mathbf{0} \end{bmatrix} \leftarrow \textit{ith vector} \quad (20)$$

where the vector on the right consists of  $m$  stacked vectors of length

$$\prod_{j=1}^t (\pi_j + \xi_j - 1)$$

where  $(\xi_1, \dots, \xi_t)$  is the order of any  $h_{ij}$ , i.e.,  $\xi_k$  is the largest power of  $z_k$  in the transform domain which has a nonzero coefficient.  $\mathbf{H}_{ij}$  is the convolution matrix determined by  $h_{ij}$  and  $(\pi_1, \dots, \pi_t)$ . The vector  $\mathbf{L}$  differs from 0 only in its first element where it is unity.  $\mathbf{G}_{ij}$  denotes the vectorized coefficients of  $g_{ij}$ . Theorem 1

gives a condition on  $h_{ij}$  for  $\mathcal{F}$  to have full rank. Equation (20) solves for the filters directly, while Equation (15) solves for the equalizers via the *determinants*  $D_i$ .

The convolution matrices  $\mathbf{H}_{ij}$  have a structure we denote as *nested Toeplitz*, with nesting level determined by the dimension  $t$  of the underlying space. For example, when  $t = 1$ ,  $\mathbf{H}_{ij}$  are Toeplitz matrices, when  $t = 2$ ,  $\mathbf{H}_{ij}$  are Toeplitz-block Toeplitz matrices and so on. Given that these matrices have special structures, it may be possible to find inverses in a computationally efficient manner. Szegő polynomials have been used for developing algorithms for the inversion of square Toeplitz-block Toeplitz matrices [3, 12, 18].

#### 4. ALMOST SURE EXISTENCE

**Definition 5** Let  $\mathcal{H}$  be as in equation (5). Let  $\phi$  denote a fixed upper bound on the total degree of the polynomials  $H_{ij}$ . Suppose that the coefficients of  $H_{ij}$  are chosen from a continuous probability density (pdf)  $\Psi$  on  $\mathbb{C}^{m\phi}$ , where

$$\Phi = \sum_{i=0}^{\phi} \binom{i+t-1}{t-1} \quad (21)$$

Then the system is said to be **random**.

**Theorem 6** Let  $\mathcal{H}$  be a matrix of size  $n \times m$ , whose elements are polynomials  $H_{ij}$  in  $t$  variables, where  $n \geq m$  and  $\binom{n}{m} > t$ . Suppose each  $H_{ij}$  is a linear combination of elements from a set  $W_{ij}$  of monomials, and that the coefficients of the linear combinations are chosen randomly from a continuous probability density function  $\Psi$  on  $\mathbb{C}^{\Omega}$ ,

$$\Omega = \sum_{i=1}^m \sum_{j=1}^n \#W_{ij}$$

where  $\#W_{ij}$  denotes the cardinality of  $W_{ij}$ . Suppose further that  $\Psi$  and  $W_{ij}$  are such that at least one of the minors  $D_k$  of size  $m \times m$  contains a nonzero constant term almost surely. Then, there almost surely exists a polynomial left inverse  $\mathcal{G}$  for  $\mathcal{H}$ .

**Proof.** Note that the coefficients of the minors  $D_k$  are polynomials of the coefficients of  $H_{ij}$ . Without loss of generality, let  $D_1$  have a nonzero constant term. Consider the homogeneous polynomials

$$E_k(z_0, z_1, \dots, z_t) = z_0^{\deg(D_k)} D_k\left(\frac{z_1}{z_0}, \frac{z_2}{z_0}, \dots, \frac{z_t}{z_0}\right) \quad (22)$$

$E_k$  share a common zero in  $\mathbb{C}^{t+1} - \{0\}$  whenever  $D_k$  share a common zero in  $\mathbb{C}^t - \{0\}$ . The resolvent  $R(\rho_1, \dots, \rho_{t+1})$  of  $t+1$  homogeneous polynomials  $\rho_1, \dots, \rho_{t+1}$  in  $t+1$  variables is a polynomial in the coefficients of  $\rho_1, \dots, \rho_{t+1}$  which vanishes iff the polynomials  $\rho_1, \dots, \rho_{t+1}$  share a common zero in  $\mathbb{C}^{t+1} - \{0\}$ . Then  $R(D_1, \dots, D_{t+1})$  is a polynomial in  $\Omega$  variables [7, p. 427, Prop. 1.1]. Hence, it vanishes only on an affine variety which is a closed and nowhere dense set of measure zero. Also, the set of coefficients where  $D_1$  vanishes at the origin is a closed linear subspace of  $\mathbb{C}^{\Omega}$  which is a nowhere dense set of measure zero. Thus, for almost every choice of the coefficients of  $H_{ij}$ , the minors  $D_k$  do not share a common zero. Hence, by Theorem 1, and since the Lebesgue integral on a set of measure zero of an atomless function vanishes, a polynomial left inverse almost surely exists. ■

**Corollary 7** Let  $\mathcal{H}$  be as in Definition 5. If  $N > t$ , and if the region of support of  $\Psi$  is not a proper subspace of  $\mathbb{C}^{\Phi}$ , the inverse filter bank almost surely exists.

Corollary 7 has been proved for the cases  $t = 1, m = 1$  [1],  $t = 1, m > 1$  [2] and  $t = 2, m = 1$  [10]. Harikumar and Bresler [8, 9] show that for  $t = 2, m = 1, n > 2$ , for almost every choice of distortion filters, there is no common zero shared by these filters. They then present an algorithm for the blind estimation of  $X_1$ . The basic idea is that if the filters do not share a common zero, then  $X_1$  must be the greatest common divisor of the  $Y_j$ .

#### 5. WIDE-BAND RADAR CALIBRATION

In polarimetric synthetic aperture radar imaging, signals are transmitted and received in two polarizations. The objective is to construct a reflectivity profile of the target for each transmitter-receiver pair. However, the nonideal nature of the antenna, system effects due to amplifiers and cabling, and effect of image processing redistribute energy in each of the transmitter-receiver pairs [6, 17]. However, the physics of the data collection and imaging imply that the energy in any pixel is smeared only over a finite neighborhood. The target responses in four channels (HH, HV, VH, VV) corresponding to each transmit and receive pair are obtained. Suppose further, by reciprocity, that the response of the target is identical in the crosspolarized (HV, VH) channels. The observed responses  $o_{ab}$  can be modeled as follows:

$$\begin{bmatrix} o_{hh} & o_{hv} \\ o_{vh} & o_{vv} \end{bmatrix} = \begin{bmatrix} r_{hh} & r_{hv} \\ r_{vh} & r_{vv} \end{bmatrix} \begin{bmatrix} s_h & s_x \\ s_x & s_v \end{bmatrix} \begin{bmatrix} t_{hh} & t_{hv} \\ t_{vh} & t_{vv} \end{bmatrix} \quad (23)$$

where  $r_{ab}$  and  $t_{ab}$  represent the receive and transmit antenna responses, and where  $s_a$  represent the response of the target. Now, assuming that these functions are polynomials, *i.e.*, FIR filters, we obtain the following system description:

$$\begin{bmatrix} o_{hh} \\ o_{hv} \\ o_{vh} \\ o_{vv} \end{bmatrix} = \begin{bmatrix} r_{hh}t_{hh} & r_{hv}t_{hh} + r_{hh}t_{vh} & r_{hv}t_{vh} \\ r_{hh}t_{hv} & r_{hv}t_{hv} + r_{hh}t_{vv} & r_{hv}t_{vv} \\ r_{vh}t_{hh} & r_{vv}t_{hh} + r_{vh}t_{vh} & r_{vv}t_{vh} \\ r_{vh}t_{hv} & r_{vv}t_{hv} + r_{vh}t_{vv} & r_{vv}t_{vv} \end{bmatrix} \begin{bmatrix} s_h \\ s_x \\ s_v \end{bmatrix} \quad (24)$$

Following the proof of Theorem 6, it can be shown that this system is invertible almost surely as

$$\binom{4}{3} = 4 > 2$$

if the antenna response functions are drawn from a continuous pdf.

When  $s_x$  is *a priori* known to be zero, *e.g.*, for a body of revolution, the system model becomes

$$\begin{bmatrix} o_{hh} \\ o_{hv} \\ o_{vh} \\ o_{vv} \end{bmatrix} = \underbrace{\begin{bmatrix} r_{hh}t_{hh} & r_{hv}t_{vh} \\ r_{hh}t_{hv} & r_{hv}t_{vv} \\ r_{vh}t_{hh} & r_{vv}t_{vh} \\ r_{vh}t_{hv} & r_{vv}t_{vv} \end{bmatrix}}_{\mathcal{H}} \begin{bmatrix} s_h \\ s_v \end{bmatrix} \quad (25)$$

and the system is invertible almost surely, *i.e.*,  $s_h$  and  $s_v$  can be recovered almost surely.

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